In this case, the optimal bank angle is still given by Eq. (12), but now with $k_l = 0$, $k_3 = k_2\, x_f$. Hence, the optimal bank angle is given by

$$k_2(x_f - x)\Delta^2 + 2u[1 + k_2 u \sin\psi]\Delta - \frac{k_2(x_f - x)}{\omega^2}(\omega^2 + u^4) = 0 \tag{19}$$

The parameters selected for the iteration are k_2 and x_f.

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A Model Reference Adaptive Control Algorithm Applied to Terminally Controlled Systems

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Introduction

FOR the past few decades, programs directed toward enhancing the air-to-ground weapon delivery accuracy of aircraft have been undertaken by the U.S. Air Force and Navy. These programs have been basically aimed at optimizing the handling qualities of the aircraft's weapon delivery and fire control systems. The underlying strategy has been to minimize the errors at release. Efforts to compensate for release mechanism errors have been pursued through analytical means.[1,2]

The main contributing factors in air-to-ground bombing dispersion are the following: positioning errors, weapon release mechanism errors, separation disturbances, bomb anomalies, wind gusts, and pilot-induced errors. It is possible, by various means, to reduce some of these errors. However, most of the dispersion will still exist under most realistic weapon release conditions. Even though laser-, infrared-, and television-guided bombs have solved a large portion of the dispersion problem, the various target acquisition and designation constraints involved in hitting a target renders their use very costly and sometimes impractical.

Missile guidance is enhanced by the movement of control surfaces that produce a response in the form of an acceleration. An adaptive scheme can be used to compare the actual vs the desired outputs.[3,4] Model reference adaptive control is very suitable for ballistic air-to-ground missiles since a predetermined trajectory is normally to be followed.[5] It should be underlined that adaptive controllers will try to keep the bomb dynamics relatively invariant with respect to the dynamic pressure.

Problem Statement

Consider a bomb released from an aircraft under "ideal" conditions, with no perturbations whatsoever and no pre- or postrelease errors. Then, the bomb will follow a fixed trajectory for a given initial velocity and attitude. The aerodynamic equations of a bomb in a ballistic trajectory are given (in two-dimensional space for convenience) by

$$\dot{v} = (1/m)\,(-D - mg\sin\gamma) \tag{1}$$

$$\dot{\gamma} = (1/mv)\,(L - mg\cos\gamma) \tag{2}$$

where v is the velocity, m the mass, D the total drag, g the acceleration due to gravity, γ the flight path angle, $(\dot{\cdot})$ the time derivative, and L the lift.

$$\ddot{\theta} = (1/J)\,(-M_{\dot\theta}\dot{\theta} - M_{\dot\alpha}\dot{\alpha}) \tag{3}$$

$$\dot{\alpha} = \dot{\theta} - \dot{\gamma} \tag{4}$$

where θ is the pitch angle, J the moment of inertia, $M_{\dot\theta}$ and $M_{\dot\alpha}$ the moment coefficients, and α the angle of attack.

The main cause of deviation of a bomb from a nominal ballistic trajectory, as defined above, is an oscillatory angle of attack primarily induced during release and separation from the aircraft by aerodynamic moments, by mechanical torques imparted by the ejection and release mechanism, and by mass variations. Such effects can be modeled as changes in the velocity components, in the mass, and in the attitude.

To formulate the problem of hitting a target with a guided bomb, the following assumptions were made:

1) There are onboard accelerometers and rate gyros that will furnish the quantities required to determine accurately the state of the bomb at given instants.

2) Control fins placed at the tail end of the bomb are moved, as required, by actuators in the tail cone.

The equations of motion with perturbations and the contributions due to the control surfaces can now be represented in the following manner:

$$\dot{v} = \frac{1}{m}\,[-D - \Delta D - (m + \Delta m)g\sin(\gamma + \Delta\gamma) - D_c] \tag{5}$$

$$\dot{\gamma} = \frac{1}{mv}\,[L + \Delta L - (m + \Delta m)g\cos(\gamma + \Delta\gamma) + L_c] \tag{6}$$

$$\ddot{\theta} = (1/J)\,(M + \Delta M + M_c) \tag{7}$$

$$\dot{\alpha} = \dot{\theta} + \Delta\dot{\theta} + \dot{\theta}_c - (\dot{\gamma} + \Delta\dot{\gamma} + \dot{\gamma}_c) \tag{8}$$

where the Δ indicate the random incremental changes that vary a few times (Δm varies only once) during a trajectory and have Gaussian distributions. Subscript c stands for the contribution due to the controls.

Now, with the initial conditions and the respective random perturbations appropriately quantified, it is not difficult to solve the ballistic trajectory of a bomb (either with or without perturbations) by means of an Adams-Moulton integration algorithm with a Runge-Kutta startup. The problem is to guide the bomb to the nominal trajectory by utilizing the measurements (assumed exact) from the available sensors and to perform the necessary corrections by deflecting the fins an appropriate amount from the release point down to impact.

Received Jan. 5, 1983; revision received June 9, 1983.

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Control Problem

In order to render software implementations for guidance and control possible, it is necessary to linearize the system equations about nominal values. Hence, a linearized version of the aerodynamic equations is generated by quasilinearization, whereby the equation for the state $x(t)$ is linearized about its previous value $x(t-\Delta t)$.[6] Thus, the linearized control problem can be expressed as

$$\dot{x}_p = A_p x_p + B_p u_p \tag{9}$$

where A_p and B_p are unknown, slowly changing matrices of appropriate dimensions and x_p is the process state vector.

When the process model of Eq. (9) is compensated by an input vector $u_p = B_k(A_k x_p + u)$, then the actual perturbed (compensated) model can be represented by the following (after going through a similar quasilinearization):

$$\dot{x}_p = (A_p + B_p B_k A_k)x_p + B_p B_k u \tag{10}$$

where subscript p means perturbed and k compensated.

$$B_p B_k = B \quad \text{and} \quad A_p B A_k = A$$

with the reference model given by

$$\dot{x} = Ax + Bu \tag{11}$$

It is possible to generate an adaptive control strategy that will best track the reference model output and minimize the performance criterion given by

$$J = x_p^T(t_f) S x_p(t_f) + \frac{1}{2}\int_0^{t_f}\left[x_p^T(t) Q x_p(t) + u^T(t) R u(t)\right]\mathrm{d}t \tag{12}$$

where t_f is the final time; S, Q, and R are weighting matrices of appropriate dimensions; and superscript T means transposition.

The control vector adaptively compensating for the variations in the process parameters will actually have the following form:

$$u_p = B_k(A_k \hat{x}_p + u) \tag{13}$$

where $\hat{x}_p$ is the estimate of the perturbed state.

The optimal control strategy will then be the following feedback control law[5]:

$$u = (B_k^{-1} R^{-1} (B_k^T)^{-1} B^T K - A_k)\hat{x}_p \tag{14}$$

where K is the positive definite solution of the following Riccati-type matrix differential equation:

$$-\dot{K} = K(A - BA_k) + (A^T - A_k^T B^T)K$$
$$- KBB_k^{-1} R^{-1} (B_k^T)^{-1} B^T K + Q \tag{15}$$

with $K(t_f) = S$.

Equation (15) has to be integrated backward in time. Furthermore, in real applications, it should be noted that $\hat{x}_p$ will have to be generated by using observers of appropriate dimension,[7] since there are typically fewer measurements than state variables.

Application to a Terminally Guided Bomb

To set up a systematic approach to the adaptive procedure outlined above, the first step was to generate the reference trajectory under the given release conditions. The state coordinates of the unperturbed path were computed and the results were stored for convenience. In the real-time guidance of bombs, both the reference and actual states have to be computed online simultaneously for synchronized comparison.

For the equations of the perturbed model, random incremental disturbances were added to both the initial state and the state of the bomb along its ballistic trajectory, such that the resulting dispersion would be comparable to existing dispersion data.

The underlying constraint in the present application is to implement guidance and control action independent of the aircraft's flight control system. The main concept is to utilize the measurements from the sensors (assumed exact in this case) and to compute the necessary control command that will cause a deflection of the control surfaces placed at the tail of the bombs, which in turn will minimize the weighted least square error between the perturbed and "ideal" states. The bombs are assumed here to travel at a fixed roll angle (in real life either roll control should be implemented or roll information should be available for appropriate attitude determination).

The weapon release conditions were taken to be as follows: $\alpha = \theta = \gamma = 0$, height above ground = 4500 ft, $v = 760 \pm 25$ fps, $\rho = 0.065$ lb/ft^3 = air density, and $\dot{\theta}_0 = 0.5 \pm 0.2$ rad/s = initial pitch rate.

The computer simulations represent 101 bomb trajectories for MK-84 bombs with perturbations. The number of bombs vs the downrange distance from the release point is plotted in Fig. 1. It was assumed that one pair of the control surfaces is

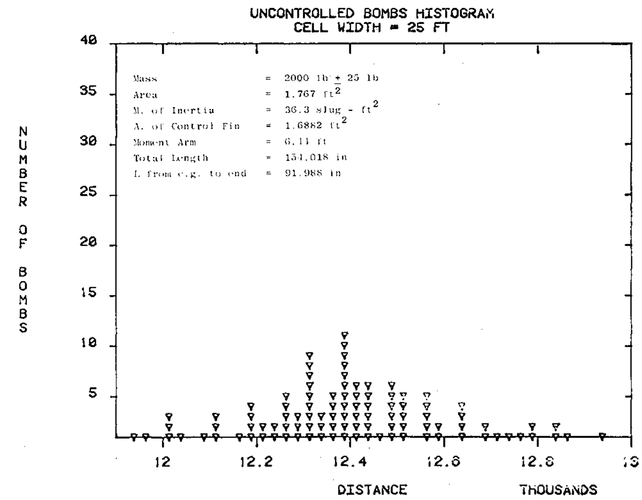

Fig. 1 Dispersion of 101 uncontrolled MK-84 bombs (cell width 25 ft).

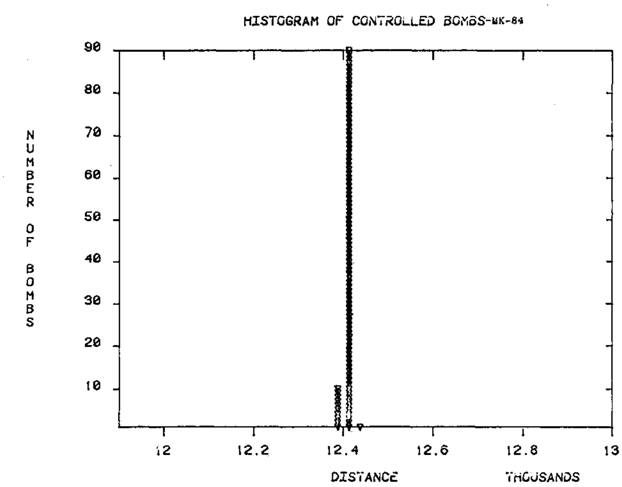

Fig. 2 Dispersion of controlled MK-84 bombs.

oriented such that the plane of its dihedral angle with the horizontal is always vertical [i.e., the control fins are in the plus (+) configuration].

The results for the MK-84 bombs with the guidance law implemented are as shown in Fig. 2. In summary, the circular error probable (CEP) for the MK-84 was 82.9 ft uncontrolled and 9.1 ft controlled. Similar simulations for the smaller MK-82 bombs resulted in comparable numbers. The CEP for the MK-82 was 79.4 ft with no control and 8.1 ft controlled.

Conclusions and Remarks

The model reference adaptive control algorithm was successfully utilized for guidance and control of iron bombs from release to impact. The results of the present simulations indicate that it is possible to terminally guide bombs with relatively inexpensive instrumentation, independent of the aircraft flight control system. Furthermore, the error of dispersion is reduced tremendously, thus creating an attractive means of solving the problem of random effects on bombs released from aircraft. The same algorithm will apply to other release conditions (such as toss or dive bombing) just as well; however, it might involve more complex mathematics and, thus, require more computations.

Acknowledgments

The various valuable suggestions of the reviewers are thankfully acknowledged.

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Use of Differential Pressure Feedback in an Automatic Flight Control System

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Introduction

THIS Note outlines a feasibility study performed to evaluate the performance of a system whereby a control surface is positioned using differential pressure across that surface as the feedback variable. In nearly all airplanes equipped with automatic flight controls, control surfaces are positioned via feedback of surface position. Since control power is often linearly related to position, this type of feedback works well. However, in many instances, it is found necessary to schedule the feedback gain with flight attitude, dynamic pressure, Mach number, or a combination thereof.

Since the purpose of any control deflection is to create a pressure differential across a surface, it is logical to consider positioning a control surface by direct feedback of this differential pressure. This method may simplify control laws. Since differential pressure is also a function of airspeed and angle of attack, it may as well allow for control of these variables.

The goal of the study documented herein was to build a simple, low-cost demonstration system and to evaluate its performance through frequency response testing. First, the pressure profile characteristics of the test surface were determined and a suitable pressure sensor arrangement selected; then closed-loop dynamic tests were conducted.

System Description

The differential pressure (ΔP) command system drives the control surface to a position such that the desired differential pressure is achieved between the two pressure sensor locations. One application is illustrated by the simplified pitch attitude hold block diagram shown in Fig. 1. The ΔP command loop merely replaces a conventional position command loop. It is the ΔP command loop that has been tested and is discussed in this Note.

Figure 2 shows the ΔP command system flow diagram. The pressure transducer selected was a commercially available piezoresistive type with a range of 0-1 psid. Its response characteristics were good, but only positive pressures could be sensed. A second sensor, with the input ports reversed, was needed to sense negative pressures. The rectifier circuit blocks the signal output by either sensor when it is below its usable range and passes the valid signal. The end result is an effective range of -1 to $+1$ psia. The signal conditioner allows for pressure or position feedback modes, lead-lag compensation if required, and monitor positions to prevent a hardover condition in the pressure feedback mode. Standard amplification methods are used to drive the actuator. The actuator is an electromechanical jack screw and is semireversible. It has a stalled output of 400 lb.

Pressure Profile Study

To determine the best location for the pressure transducer, baseline data were obtained through wind-tunnel testing on the pressure distribution around the airfoil as a function of angle of attack and flap deflection. All wind-tunnel tests were in the University of Kansas 3 × 4 ft low-speed wind tunnel and are completely documented separately.[1] Angle of attack α and flap deflection δ limits were ± 8 and ± 20 deg, respectively. The static pressures at 13 chordwise locations on both sides of the surface were read from a slant-tube manometer board for each combination of α and δ. After applicable corrections were made, the data were reduced to differential pressure coefficient form,

$$\Delta C_P = (P_{\text{lower}} - P_{\text{upper}})/\bar{q} \tag{1}$$

The differential pressure at each chordwise location is found to be a linear function of angle of attack and flap deflection. The data can be further reduced by deriving the following coefficients:

$$C_{P_\alpha} = \frac{\partial(\Delta C_P)}{\partial \alpha}; \qquad C_{P_\delta} = \frac{\partial(\Delta C_P)}{\partial \delta} \tag{2}$$

Submitted Aug. 9, 1982; revision received July 25, 1983.

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